GW approximation in exciting

exciting

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HoW exciting! 2023

4th August



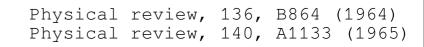
Outline

Introduction

- Why GW (beyond DFT)
- GW approximation
- Implementation in exciting
- How to use GW@exciting
 - Input file preparation
 - Output file inspection
 - Convergence test
- Summary

Kohn-Sham equation

$$\left\{-\frac{1}{2}\nabla^2 + v_{\rm ne}(\mathbf{r}) + v_{\rm H}(\mathbf{r}) + v_{\rm xc}(\mathbf{r})\right\}\psi_i^{KS}(\mathbf{r}) = \epsilon_i^{KS}\psi_i^{KS}(\mathbf{r})$$



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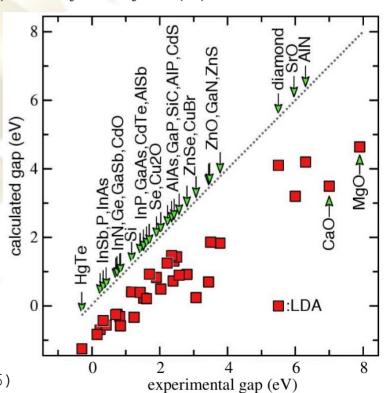
 DFT does not give any recipe for exchange-correlation potential (LDA, GGA, Hybrid ...)

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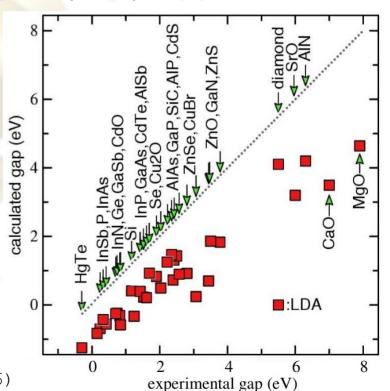


Kohn-Sham equation

$$-\frac{1}{2}\nabla^2 + v_{\rm ne}(\mathbf{r}) + v_{\rm H}(\mathbf{r}) + v_{\rm xc}(\mathbf{r}) \bigg\} \psi_i^{KS}(\mathbf{r}) = \epsilon_i^{KS} \psi_i^{KS}(\mathbf{r})$$

- DFT does not give any recipe for exchange-correlation potential (LDA, GGA, Hybrid ...)
- Ground-state formalism
- Predicts GS energy
- Inadequate description of quasi-particle nature of electron

Schilfgaarde et al. PRL 96 226402(2006)



Kohn-Sham equation

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Quasi-particle equation

$$\left\{-\frac{1}{2}\nabla^2 + v_{\rm ne}(\mathbf{r}) + v_{\rm H}(\mathbf{r})\right\}\psi_i^{QP}(\mathbf{r}) + \int d\mathbf{r}' \Sigma\left(\mathbf{r}, \mathbf{r}', \epsilon_i^{QP}\right)\psi_i^{QP}(\mathbf{r}') = \epsilon_i^{QP}\psi_i^{QP}(\mathbf{r})$$

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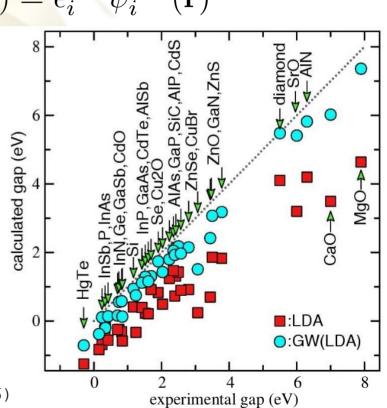
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Schilfgaarde et al. PRL 96 226402(2006)



Kohn-Sham equation

$$\left[-\frac{1}{2}\nabla^{2}+v_{\mathrm{ne}}(\mathbf{r})+v_{\mathrm{H}}(\mathbf{r})+v_{\mathrm{xc}}(\mathbf{r})\right]\psi_{i}^{KS}(\mathbf{r})=\epsilon_{i}^{KS}\psi_{i}^{KS}(\mathbf{r})$$

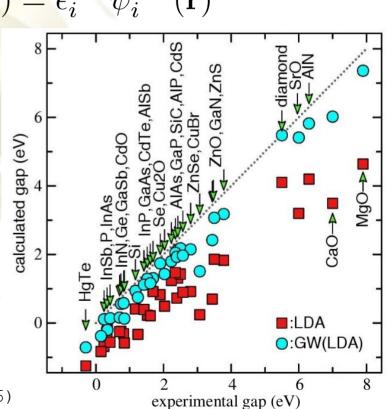
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► How to calculate the **self-energy**?

Schilfgaarde et al. PRL 96 $2264\overline{02}$ (2006)



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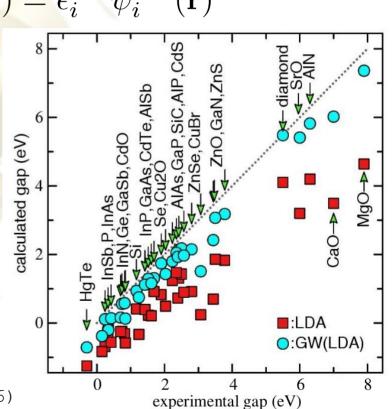
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- ▶ How to calculate the **self-energy**?
- Hedin's equations $\Sigma = GW$

Schilfgaarde et al. PRL 96 226402 (2006)



Self-energy

$$\Sigma(\mathbf{r},\mathbf{r}',\omega) = \frac{i}{2\pi} \int G(\mathbf{r},\mathbf{r}',\omega-\omega') W(\mathbf{r},\mathbf{r}',\omega') e^{-i\delta\omega'} d\omega'$$

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Screened Coulomb interaction

$$W(\mathbf{r}, \mathbf{r}', \omega) = \int \epsilon^{-1} (\mathbf{r}, \mathbf{r}'', \omega) v(\mathbf{r}'', \mathbf{r}') d\mathbf{r}''$$

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Dielectric function

$$\epsilon(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') - \int v(\mathbf{r}, \mathbf{r}'') P(\mathbf{r}'', \mathbf{r}', \omega) d\mathbf{r}''$$

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Dielectric function

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Polarizability function

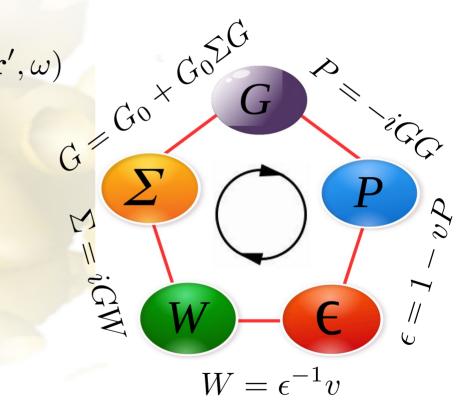
$$P(\mathbf{r}, \mathbf{r}', \omega) = -\frac{i}{2\pi} \int G(\mathbf{r}', \mathbf{r}, \omega - \omega') G(\mathbf{r}, \mathbf{r}', \omega') e^{-i\delta\omega'} d\omega'$$

Green's function (updated)

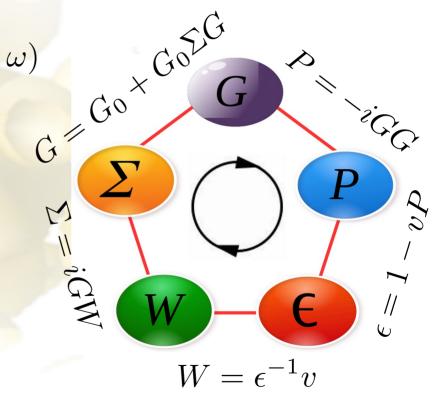
$$G^{-1}(\mathbf{r}, \mathbf{r}', \omega) = G_0^{-1}(\mathbf{r}, \mathbf{r}', \omega) - \Sigma(\mathbf{r}, \mathbf{r}', \omega)$$

Green's function (updated)

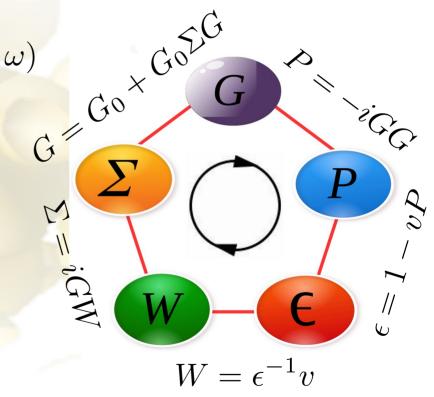
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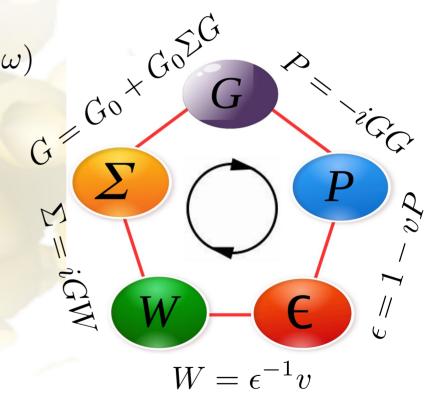
- Green's function (updated) $G^{-1}(\mathbf{r}, \mathbf{r}', \omega) = G_0^{-1}(\mathbf{r}, \mathbf{r}', \omega) - \Sigma(\mathbf{r}, \mathbf{r}', \omega)$
- Quasi-particle energies $\epsilon_{n\mathbf{k}}^{QP} = \epsilon_{n\mathbf{k}}^{KS} + \langle \psi_{n\mathbf{k}}^{KS} \mid \Sigma - V_{xc}^{KS} \mid \psi_{n\mathbf{k}}^{KS} \rangle$



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- Green's function from KS states $G_0(\mathbf{r}, \mathbf{r}', \omega) = \sum_{n\mathbf{k}} \frac{\psi_{n\mathbf{k}}^{KS}(\mathbf{r})\psi_{n\mathbf{k}}^{KS^*}(\mathbf{r}')}{\omega - \epsilon_{n\mathbf{k}}^{KS} \pm i\eta}$



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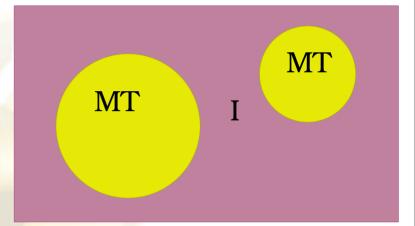






- All electron DFT code
- Employs LAPW and LO
- Treats core electron with radial Dirac eq.
- ▶ DFT, TDDFT (RTD), GW, BSE ...

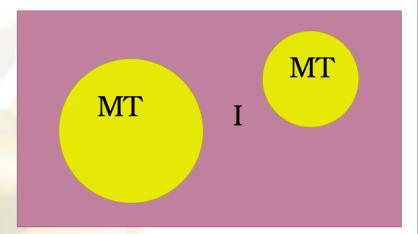


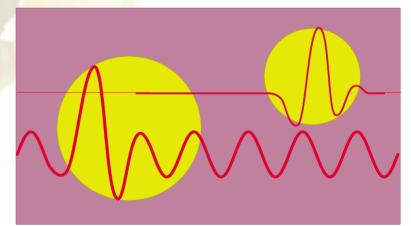


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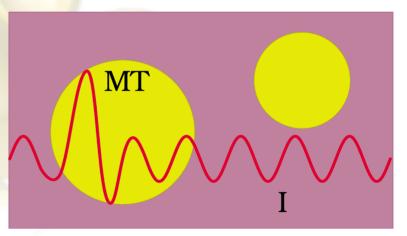




https://exciting-code.org/

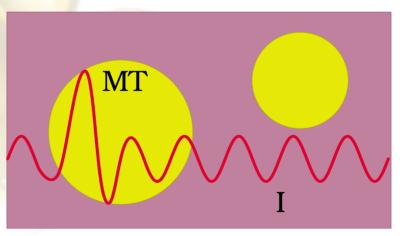
Linearized augmented plane wave (LAPW) basis

$$\phi_{\mathbf{G}}^{\mathbf{k}}(\mathbf{r}) = \begin{cases} \sum_{lm} \left[A_{\alpha lm} (\mathbf{k} + \mathbf{G}) u_{\alpha l} \left(r^{\alpha}, E_{l} \right) + \dots \right] Y_{lm} \left(\hat{\mathbf{r}}^{\alpha} \right) & r^{\alpha} < R_{\mathrm{MT}}^{\alpha} \\ \frac{1}{\sqrt{\Omega}} e^{i(\mathbf{k} + \mathbf{G}) \cdot \mathbf{r}} & \mathbf{r} \in I \end{cases}$$



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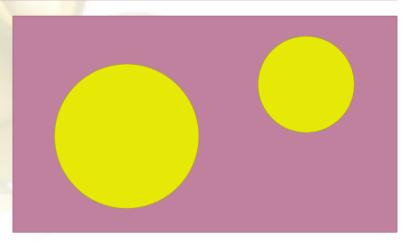
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• Kohn-Sham wave function $\psi_{n\mathbf{k}}^{KS}(\mathbf{r}) = \sum_{\mathbf{r}} C_{n\mathbf{k},\mathbf{G}} \phi_{\mathbf{G}}^{\mathbf{k}}(\mathbf{r})$



A. Gulans et al. J. Phys. Cond. Mat. 26(36), 363202 (2014)

- Mixed basis in MT region
 - We only consider $u_{\alpha l}(r^{\alpha})$ with $l \leq l_{max}^{MB}$

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• We take $u_{\alpha l}(r^{\alpha})u_{\alpha l'}(r^{\alpha}) = v_{\alpha NL}(r^{\alpha})$ with $|l - l'| \leq L \leq |l + l'|$

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 - Overlap matrix, diagonalize and eliminate linearly dependent ones

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epsmb

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 - Overlap matrix, diagonalize and eliminate linearly dependent ones
 - Product basis with translational symmetry

$$\gamma_{\alpha NLM}^{\mathbf{q}}(\mathbf{r}) = \frac{1}{\sqrt{N_c}} \sum_{\mathbf{R}} e^{i\mathbf{q}\cdot(\mathbf{r}_{\alpha} + \mathbf{R})} v_{\alpha NL} \left(r^{\alpha}\right) Y_{LM} \left(\hat{\mathbf{r}}^{\alpha}\right)$$

F. Aryasetiawan and O. Gunnarsson, Phys. Rev. B 49, 16214 (1994)T. Kotani and M. van Schilfgaarde, Solid State Comm. 121, 461 (2002)

epsmb

- Mixed basis in Interstitial region
 - Overlap matrix $\mathbb{O}_{\mathbf{G}\mathbf{G}'} = \frac{1}{\Omega} \int_{\Omega} \theta_{I}(\mathbf{r}) e^{i(\mathbf{G}-\mathbf{G}')\cdot\mathbf{r}} d^{3}r$

- Mixed basis in Interstitial region
 - Overlap matrix

$$\mathbb{O}_{\mathbf{G}\mathbf{G}'} = \frac{1}{\Omega} \int_{\Omega} \theta_I(\mathbf{r}) e^{i(\mathbf{G}-\mathbf{G}')\cdot\mathbf{r}} d^3r$$

Diagonalize the overlap matrix

$$\sum_{\mathbf{G}'} \mathbb{O}_{\mathbf{G}\mathbf{G}'} S_{\mathbf{G}'i} = \lambda_i S_{\mathbf{G}i}$$

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Orthonormal basis set

$$P_i^{\mathbf{q}}(\mathbf{r}) \equiv \frac{1}{\sqrt{\Omega}} \sum_{\mathbf{G}}^{G_{max}^{MB}} \tilde{S}_{\mathbf{G}i} e^{i(\mathbf{G}+\mathbf{q})\cdot\mathbf{r}} \theta_I(\mathbf{r})$$

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Polarizability

$$P_{0}(\mathbf{r},\mathbf{r}',\omega) = \sum_{n,m} \sum_{\mathbf{k},\mathbf{q}} F_{nm}(\mathbf{k},\mathbf{q};\omega) \psi_{n\mathbf{k}}(\mathbf{r}) \psi_{m\mathbf{k}-\mathbf{q}}^{*}(\mathbf{r}) \psi_{n\mathbf{k}}^{*}(\mathbf{r}') \psi_{m\mathbf{k}-\mathbf{q}}(\mathbf{r}')$$

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$$\psi_{n\mathbf{k}}(\mathbf{r})\psi_{m\mathbf{k}-\mathbf{q}}^{*}(\mathbf{r}) = \sum_{i} M_{nm}^{i}(\mathbf{k},\mathbf{q})\chi_{i}^{\mathbf{q}}(\mathbf{r})$$

Orthonormal mixed basis set

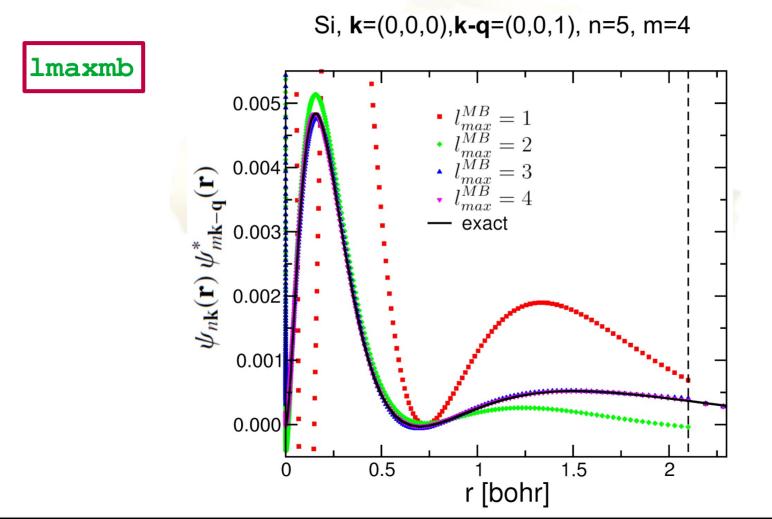
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Polarizability

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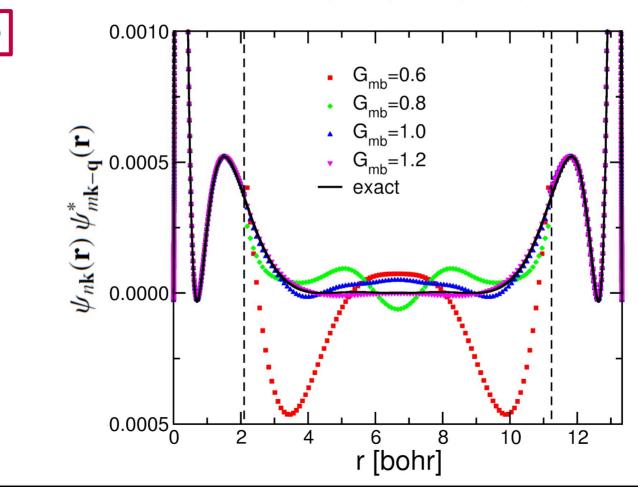
$$\psi_{n\mathbf{k}}(\mathbf{r})\psi_{m\mathbf{k}-\mathbf{q}}^{*}(\mathbf{r}) = \sum_{i} M_{nm}^{i}(\mathbf{k},\mathbf{q})\chi_{i}^{\mathbf{q}}(\mathbf{r})$$

$$M_{nm}^{i}(\mathbf{k},\mathbf{q}) \equiv \int_{\Omega} \left[\chi_{i}^{\mathbf{q}}(\mathbf{r})\psi_{m\mathbf{k}-\mathbf{q}}(\mathbf{r})\right]^{*}\psi_{n\mathbf{k}}(\mathbf{r})d\mathbf{r}$$



gmb

Si, **k**=(0,0,0),**k-q**=(0,0,1), n=5, m=4



Self-energy can be split into two parts

$$\Sigma(\mathbf{r}, \mathbf{r}', \omega) = \Sigma^{x}(\mathbf{r}, \mathbf{r}') + \Sigma^{c}(\mathbf{r}, \mathbf{r}', \omega)$$

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$$\Sigma\left(\mathbf{r},\mathbf{r}',\omega
ight)=\Sigma^{x}\left(\mathbf{r},\mathbf{r}'
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Exchange part of the self-energy

$$\Sigma_{n\mathbf{k}}^{x} = -\frac{1}{N_{c}} \sum_{\mathbf{q}}^{BZ} \sum_{i,j} v_{ij}(\mathbf{q}) \sum_{m}^{occ} \left[M_{nm}^{i}(\mathbf{k},\mathbf{q}) \right]^{*} M_{nm}^{j}(\mathbf{k},\mathbf{q})$$

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ight)+\Sigma^{c}\left(\mathbf{r},\mathbf{r}',\omega
ight)$$

Exchange part of the self-energy

$$\Sigma_{n\mathbf{k}}^{x} = -\frac{1}{N_{c}} \sum_{\mathbf{q}}^{BZ} \sum_{i,j} v_{ij}(\mathbf{q}) \sum_{m}^{occ} \left[M_{nm}^{i}(\mathbf{k},\mathbf{q}) \right]^{*} M_{nm}^{j}(\mathbf{k},\mathbf{q})$$

Coulomb potential in the product basis

$$v_{ij}(\mathbf{q}) = \int_{\Omega} \int_{\Omega} \left[\chi_i^{\mathbf{q}}(\mathbf{r}) \right]^* v(\mathbf{r}, \mathbf{r}') \chi_j^{\mathbf{q}}(\mathbf{r}') d\mathbf{r} d\mathbf{r}'$$

Polarizability in mixed basis

 $P_{ij}(\mathbf{q},\omega) = \frac{1}{N_c} \sum_{\mathbf{k}}^{BZ} \sum_{n}^{occ} \sum_{m}^{empty} F_{nm}(\mathbf{k},\mathbf{q};\omega) M_{nm}^i(\mathbf{k},\mathbf{q}) \left[M_{nm}^j(\mathbf{k},\mathbf{q}) \right]^*$

Polarizability in mixed basis

 $P_{ij}(\mathbf{q},\omega) = \frac{1}{N_c} \sum_{\mathbf{k}}^{BZ} \sum_{n}^{occ \ empty} \sum_{m}^{P_{nm}(\mathbf{k},\mathbf{q};\omega)} M_{nm}^{i}(\mathbf{k},\mathbf{q}) \left[M_{nm}^{j}(\mathbf{k},\mathbf{q}) \right]^{*}$

ngridq

Polarizability in mixed basis

$P_{ij}(\mathbf{q},\omega) = \frac{1}{N_c} \sum_{\mathbf{k}}^{BZ} \sum_{n}^{occ} \sum_{m}^{empty} F_{nm}(\mathbf{k},\mathbf{q};\omega) M_{nm}^i(\mathbf{k},\mathbf{q}) \left[M_{nm}^j(\mathbf{k},\mathbf{q}) \right]^*$



Polarizability in mixed basis

$P_{ij}(\mathbf{q}\,\boldsymbol{\omega}) = \frac{1}{N_c} \sum_{\mathbf{k}}^{BZ} \sum_{n}^{occ} \sum_{m}^{empty} F_{nm}(\mathbf{k}, \mathbf{q}; \boldsymbol{\omega}) M_{nm}^i(\mathbf{k}, \mathbf{q}) \left[M_{nm}^j(\mathbf{k}, \mathbf{q}) \right]^*$

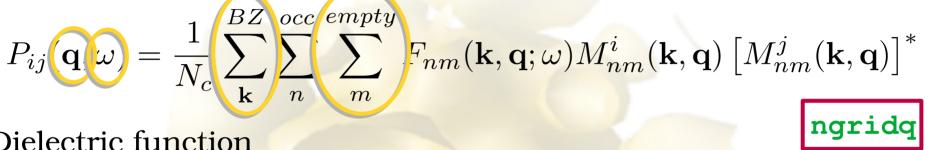


Polarizability in mixed basis

Dielectric function

$$\epsilon_{ij}\left(\mathbf{q},\omega\right) = \delta_{ij} - \sum_{i'j'} v_{ii'}^{\frac{1}{2}}\left(\mathbf{q}\right) P_{i'j'}\left(\mathbf{q},\omega\right) v_{j'j}^{\frac{1}{2}}\left(\mathbf{q}\right)$$

Correlation part of the screened Coulomb interaction $W_{ij}^{c}\left(\mathbf{q},\omega\right) = \sum v_{ii'}^{\frac{1}{2}}\left(\mathbf{q}\right) \left[\epsilon_{i'j'}^{-1}\left(\mathbf{q},\omega\right) - \delta_{i'j'}\right] v_{j'j}^{\frac{1}{2}}\left(\mathbf{q}\right)$



nempty

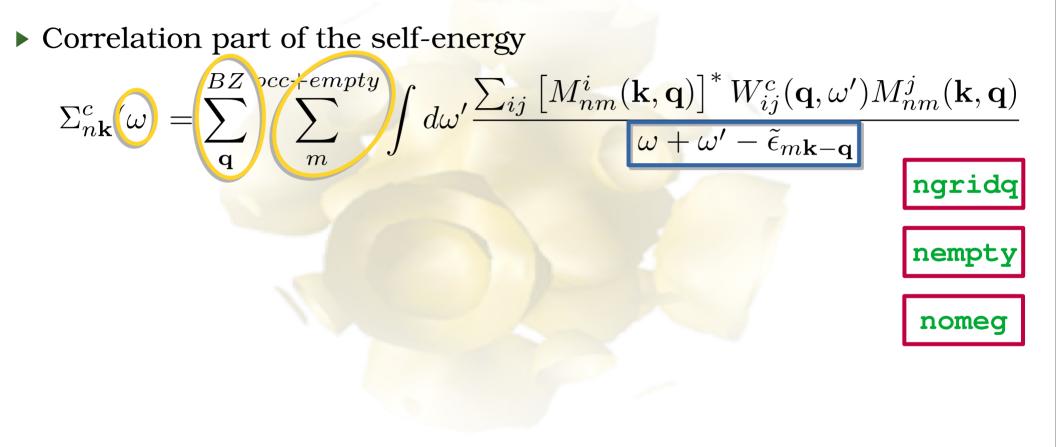
nomeg

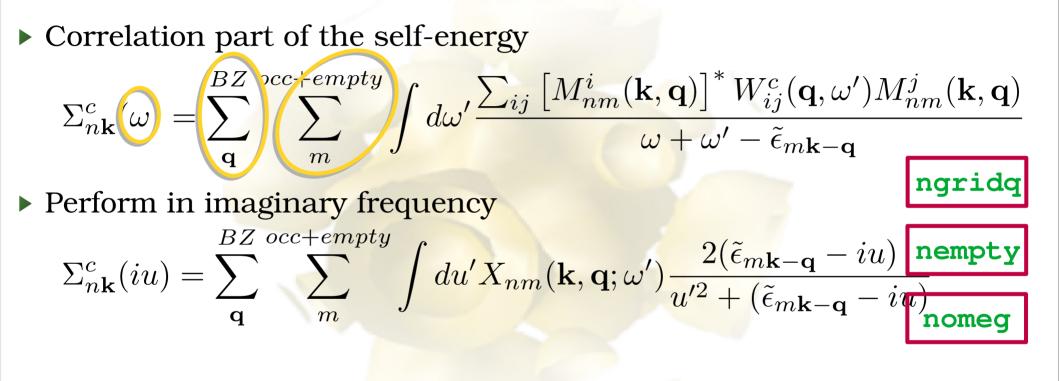
Correlation part of the self-energy

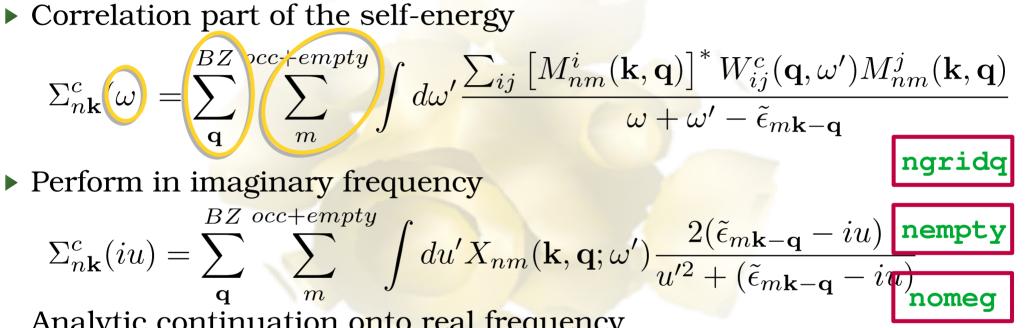
$$\Sigma_{n\mathbf{k}}^{c}(\omega) = \sum_{\mathbf{q}}^{BZ} \sum_{m}^{occ+empty} \int d\omega' \frac{\sum_{ij} \left[M_{nm}^{i}(\mathbf{k},\mathbf{q}) \right]^{*} W_{ij}^{c}(\mathbf{q},\omega') M_{nm}^{j}(\mathbf{k},\mathbf{q})}{\omega + \omega' - \tilde{\epsilon}_{m\mathbf{k}-\mathbf{q}}}$$



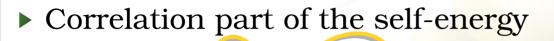
$$\Sigma_{n\mathbf{k}}^{c}\omega = \sum_{\mathbf{q}}^{BZ}\sum_{m}^{cc\text{,rempty}}\int d\omega' \frac{\sum_{ij} \left[M_{nm}^{i}(\mathbf{k},\mathbf{q})\right]^{*}W_{ij}^{c}(\mathbf{q},\omega')M_{nm}^{j}(\mathbf{k},\mathbf{q})}{\omega + \omega' - \tilde{\epsilon}_{m\mathbf{k}-\mathbf{q}}} \frac{\text{ngridq}}{\text{nempty}}$$







Analytic continuation onto real frequency



Perform in imaginary frequency

$$\Sigma_{n\mathbf{k}}^{c}(iu) = \sum_{\mathbf{q}}^{BZ} \sum_{m}^{occ+empty} \int du' X_{nm}(\mathbf{k},\mathbf{q};\omega') \frac{2(\tilde{\epsilon}_{m\mathbf{k}-\mathbf{q}}-iu)}{u'^{2}+(\tilde{\epsilon}_{m\mathbf{k}-\mathbf{q}}-iu)} \frac{\mathsf{nempty}}{\mathsf{nomeg}}$$

 $\Sigma_{n\mathbf{k}}^{c}\omega = \sum_{i=1}^{BZ} \sum_{j=1}^{DCC+empty} \int d\omega' \frac{\sum_{ij} \left[M_{nm}^{i}(\mathbf{k},\mathbf{q})\right]^{*} W_{ij}^{c}(\mathbf{q},\omega') M_{nm}^{j}(\mathbf{k},\mathbf{q})}{\omega + \omega' - \tilde{\epsilon}_{m\mathbf{k}-\mathbf{q}}}$

Analytic continuation onto real frequency

Quasi-particle energies

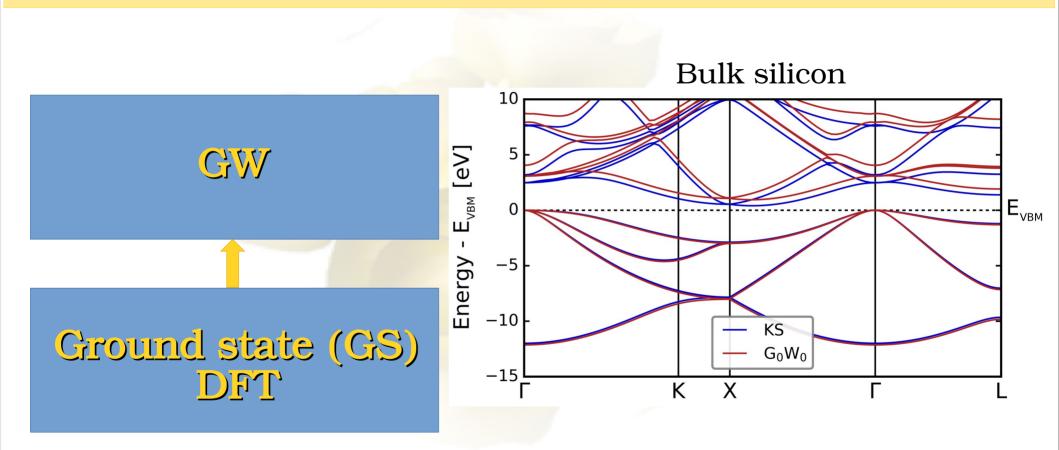
$$\epsilon_{n\mathbf{k}}^{QP} = \epsilon_{n\mathbf{k}}^{KS} + Z_{n\mathbf{k}} \left[\mathcal{R}\Sigma_{n\mathbf{k}}^{c}(\epsilon_{n\mathbf{k}}^{KS}) + \Sigma_{n\mathbf{k}}^{x} - V_{xc}^{KS} \right]$$

H. Jiang et al., Comput. Phys. Commun. 184, 348 (2013)

ngridq

Cround state (CS) DFT





► GS input file

► GS input file

<input>

<title>Silicon-GW</title>

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<structure

speciespath="/users/sol/manoarphy/neon/species">
    <crystal>
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        <basevect>0.00 5.13 5.13</basevect>
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        <br/>
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        </species>
</structure>
```

```
<groundstate
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    ngridk="3 3 3"
    xctype="LDA_PW"
></groundstate>
```

</input>

► GS input file



<title>Silicon-GW</title>

```
<structure

speciespath="/users/sol/manoarphy/neon/species">

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<basevect>5.13 5.13 0.00</basevect>

<basevect>5.13 0.00 5.13</basevect>

<basevect>0.00 5.13 5.13</basevect>

</crystal>

<species speciesfile="Si.xml" rmt="2.1">

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</species>

</structure>
```

```
<groundstate
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    rgkmax="7.0"
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    xctype="LDA_PW"
></groundstate>
```



► GS input file



<title>Silicon-GW</title>

<structure< th=""></structure<>
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<pre><basevect>5.13 0.00 5.13</basevect></pre> /basevect>
<pre><basevect>0.00 5.13 5.13</basevect></pre>
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<pre><atom coord="0.00 0.00 0.00"></atom></pre>
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► GS input file



<title>Silicon-GW</title>

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► GS input file



<title>Silicon-GW</title>

<pre><structure speciespath="/users/sol/manoarphy/neon/species"> <crystal></crystal></structure></pre>		
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<pre><species rmt="2.1" speciesfile="Si.xml"></species></pre>		
<atom coord="0.00 0.00 0.00"></atom>		
<atom coord="0.25 0.25 0.25"></atom>		



► GS input file



<title>Silicon-GW</title>

<structure< th=""></structure<>
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<pre><species rmt="2.1" speciesfile="Si.xml"></species></pre>
<atom coord="0.00 0.00 0.00"></atom>
<atom coord="0.25 0.25 0.25"></atom>



► GS input file



<title>Silicon-GW</title>

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► GS input file



<title>Silicon-GW</title>

```
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<basevect>0.00 5.13 5.13</basevect>

</crystal>

<species speciesfile="Si.xml" rmt="2.1">

<atom coord="0.00 0.00 0.00"></atom>

<atom coord="0.25 0.25 0.25"></atom>

</species>

</structure>
```

```
<groundstate
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    rgkmax="7.0"
    ngridk="3 3 3"
    xctype="LDA_PW"
></groundstate>
```



► GS input file



<title>Silicon-GW</title>

```
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speciespath="/users/sol/manoarphy/neon/species">
    <crystal>
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        <basevect>0.00 5.13 5.13</basevect>
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        </crystal>
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            <atom coord="0.00 0.00 0.00"></atom>
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        </species>
</structure>
```

```
<groundstate
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```



► GW input file

► GW input file

<input>

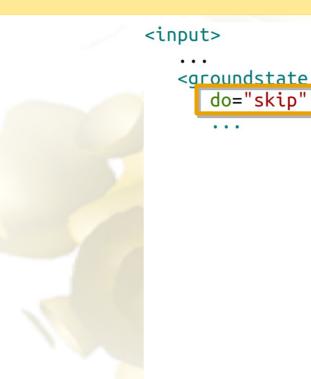
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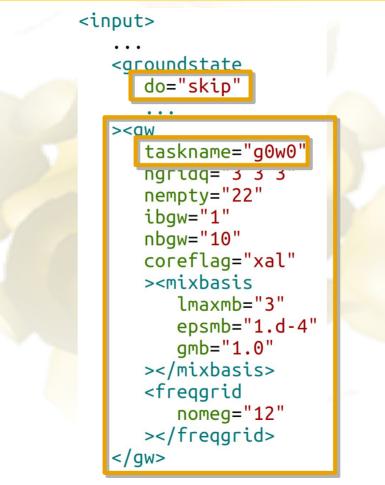
► GW input file



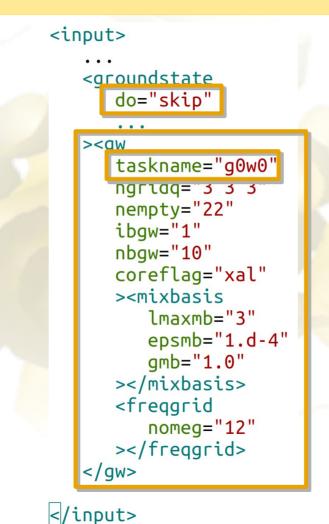




► GW input file

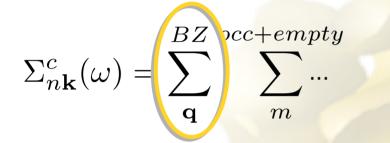


► GW input file



taskname="band"
taskname="dos"

► GW input file



<input> . . . <groundstate do="skip" . . . ><gw taskname="00w0" ngridq="3 3 3" nempty="22" ibgw="1" nbgw="10" coreflag="xal" ><mixbasis</pre> lmaxmb="3" epsmb="1.d-4" gmb="1.0" ></mixbasis> <freqgrid nomeg="12" ></freqgrid> </gw>

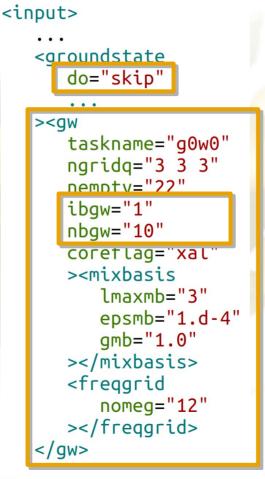
► GW input file



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do="skip"
> <gw< td=""></gw<>
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nbgw="10"
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epsmb="1.d-4"
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>
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>
1.5

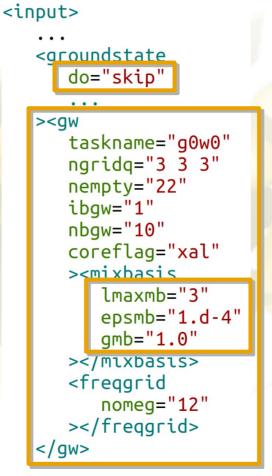
► GW input file

 $BZ \ occ + empty$ $\Sigma_{n_{\mathbf{c}}}(\omega) = \sum_{\mathbf{q}} \sum_{m} \dots$

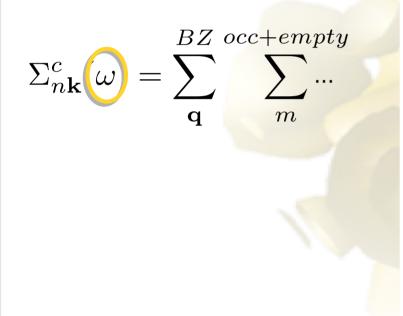


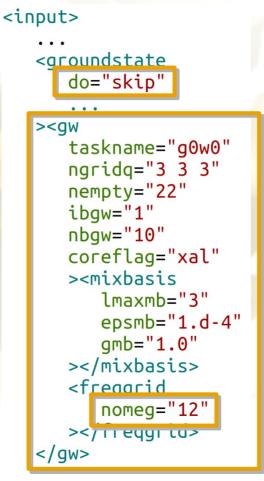
► GW input file

$$\Sigma_{n\mathbf{k}}^{c}(\omega) = \sum_{\mathbf{q}}^{BZ} \sum_{m}^{occ+empty} \dots$$



► GW input file





atoms.xml BANDLINES.OUT BAND.OUT BAND-OP.OUT bandstructure.dat bandstructure-gp.dat bandstructure-qp.xml bandstructure.xml BONDLENGTH.OUT dos.xml EFERMI GW.OUT EFERMI.OUT EIGVAL.OUT eiqval.xml

EPS00 GW.OUT EOATOMS.OUT EVALCORE.OUT evalcore.xml EVALFV GW.OUT EVALFV.OUT EVALQP.DAT EVALOP.OUT EVALSV GW.OUT EVALSV.OUT EVECFV_GW.OUT EVECFV.OUT EVECSV GW.OUT EVECSV.OUT geometry.xml GW INFO.OUT

INF0.OUT info.xml input.xml KPOINTS.OUT LATTICE.OUT LINENGY.OUT OCCSV.OUT PLOT.eps PMATVV.OUT RMSDVEFF.OUT SELFC.OUT SELFX.OUT si-gw-dos-KS.GW.png si-gw-ef.png si-gw-vbm.png Si ini.xml

Si_scf.xml STATE.OUT SYMCRYS.OUT SYMGENR.OUT SYMINV.OUT SYMLAT.OUT SYMMULT.OUT SYMSITE.OUT SYMT2.OUT TDOS.OUT TDOS-OP.OUT TOTENERGY.OUT VXCNN.DAT VXCNN.OUT

•

atoms.xml BANDLINES.OUT BAND.OUT BAND-OP.OUT bandstructure.dat bandstructure-gp.dat bandstructure-qp.xml bandstructure.xml BONDLENGTH.OUT dos.xml EFERMI GW.OUT EFERMI.OUT EIGVAL.OUT eiqval.xml

EPS00 GW.OUT EOATOMS.OUT EVALCORE.OUT evalcore.xml EVALFV GW.OUT EVALFV.OUT EVALQP.DAT EVALOP.OUT EVALSV GW.OUT EVALSV.OUT EVECFV_GW.OUT EVECFV.OUT EVECSV GW.OUT EVECSV.OUT geometrv.xml GW INFO.OUT

INF0.OUT info.xml input.xml KPOINTS.OUT LATTICE.OUT LINENGY.OUT OCCSV.OUT PLOT.eps PMATVV.OUT RMSDVEFF.OUT SELFC.OUT SELFX.OUT si-gw-dos-KS.GW.png si-gw-ef.png si-gw-vbm.png Si ini.xml

Si_scf.xml STATE.OUT SYMCRYS.OUT SYMGENR.OUT SYMINV.OUT SYMLAT.OUT SYMMULT.OUT SYMSITE.OUT SYMT2.OUT TDOS.OUT TDOS-OP.OUT TOTENERGY.OUT VXCNN.DAT VXCNN.OUT

atoms.xml **BANDLINES.OUT** BAND.OUT BAND-OP.OUT bandstructure.dat bandstructure-gp.dat bandstructure-qp.xml bandstructure.xml BONDLENGTH.OUT dos.xml EFERMI GW.OUT EFERMI.OUT EIGVAL.OUT eiqval.xml

EPS00 GW.OUT EOATOMS.OUT EVALCORE.OUT evalcore.xml EVALFV GW.OUT EVALFV.OUT EVALQP.DAT EVALOP.OUT EVALSV GW.OUT EVALSV.OUT EVECFV_GW.OUT EVECFV.OUT EVECSV GW.OUT EVECSV.OUT geometry.xml GW INFO.OUT

INF0.OUT info.xml input.xml KPOINTS.OUT LATTICE.OUT LINENGY.OUT OCCSV.OUT PLOT.eps PMATVV.OUT RMSDVEFF.OUT SELFC.OUT SELFX.OUT si-gw-dos-KS.GW.png si-gw-ef.png si-gw-vbm.png Si ini.xml

Si_scf.xml STATE.OUT SYMCRYS.OUT SYMGENR.OUT SYMINV.OUT SYMLAT.OUT SYMMULT.OUT SYMSITE.OUT SYMT2.OUT TDOS.OUT TDOS-OP.OUT TOTENERGY.OUT VXCNN.DAT VXCNN.OUT

k-point	t# 1:	0.000000	0.00000	0.00000	0 0.0370	937				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
k point	⊾# ⊃.	0 000000	0 00000	0 22222	2 0 2067	206				
k-point		0.000000	0.000000				Nh e			7-1/
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.40055	-0.62733	-0.40035	-0.69022	0.22708	0.02707	-0.46344	-0.22678	0.00020	0.68318
2	-0.20836	-0.33803	-0.20833	-0.58997	0.12970	0.00745	-0.46029	-0.12968	0.00002	0.74643
3	-0.04760	-0.12044	-0.04295	-0.55914	0.07884	0.00053	-0.48631	-0.07284	0.00466	0.77473
4	-0.04760	-0.12044	-0.04300	-0.55914	0.07879	0.00050	-0.48631	-0.07284	0.00460	0.77383
5	0.04899	0.22993	0.07841	-0.28110	-0.14333	-0.00053	-0.46204	0.18094	0.02942	0.78220
6	0.11512	0.30416	0.14384	-0.22532	-0.15215	-0.00174	-0.41437	0.18905	0.02872	0.77830
7	0.11512	0.30416	0.14394	-0.22532	-0.15203	-0.00165	-0.41437	0.18905	0.02883	0.77885
8	0.26948	0.49261	0.29430	-0.15957	-0.19119	-0.00829	-0.38270	0.22314	0.02483	0.77719
9 10	0.28052	0.50287	0.31461	-0.16878	-0.17908	-0.00819	-0.39114	0.22236	0.03409	0.78765

k-point		0.000000	0.00000	0.0000						
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
k-point	# 2:	0.000000	0.000000	0.33333	0.2962	296				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.40055	-0.62733	-0.40035	-0.69022	0.22708	0.02707	-0.46344	-0.22678	0.00020	0.68318
2	-0.20836	-0.33803	-0.20833	-0.58997	0.12970	0.00745	-0.46029	-0.12968	0.00002	0.74643
3	-0.04760	-0.12044	-0.04295	-0.55914	0.07884	0.00053	-0.48631	-0.07284	0.00466	0.77473
4	-0.04760	-0.12044	-0.04300	-0.55914	0.07879	0.00050	-0.48631	-0.07284	0.00460	0.77383
5	0.04899	0.22993	0.07841	-0.28110	-0.14333	-0.00053	-0.46204	0.18094	0.02942	0.78220
6	0.11512	0.30416	0.14384	-0.22532	-0.15215	-0.00174	-0.41437	0.18905	0.02872	0.77830
7	0.11512	0.30416	0.14394	-0.22532	-0.15203	-0.00165	-0.41437	0.18905	0.02883	0.77885
8	0.26948	0.49261	0.29430	-0.15957	-0.19119	-0.00829	-0.38270	0.22314	0.02483	0.77719
9	0.28052	0.50287	0.31461	-0.16878	-0.17908	-0.00819	-0.39114	0.22236	0.03409	0.78765
10	0.28052	0.50287	0.31450	-0.16878	-0.17921	-0.00835	-0.39114	0.22236	0.03398	0.78768

k-point	# 1:	0.000000	0.000000	0.0000	0.037	937				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
	9									
k-point		0.000000	0.000000							
k-point state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
-	E_KS -0.40055	E_HF -0.62733	E_GW -0.40035	Sx -0.69022	Re(Sc) 0.22708	Im(Sc) 0.02707	-0.46344	-0.22678	0.00020	0.68318
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)				
state 1	E_KS -0.40055	E_HF -0.62733	E_GW -0.40035	Sx -0.69022	Re(Sc) 0.22708	Im(Sc) 0.02707	-0.46344	-0.22678	0.00020	0.68318
state 1 2	E_KS -0.40055 -0.20836	E_HF -0.62733 -0.33803	E_GW -0.40035 -0.20833	Sx -0.69022 -0.58997	Re(Sc) 0.22708 0.12970	Im(Sc) 0.02707 0.00745	-0.46344 -0.46029	-0.22678 -0.12968	0. <u>0</u> 0020 0.00002	0.68318 0.74643
state 1 2 3	E_KS -0.40055 -0.20836 -0.04760	E_HF -0.62733 -0.33803 -0.12044	E_GW -0.40035 -0.20833 -0.04295	Sx -0.69022 -0.58997 -0.55914	Re(Sc) 0.22708 0.12970 0.07884	Im(Sc) 0.02707 0.00745 0.00053	-0.46344 -0.46029 -0.48631	-0.22678 -0.12968 -0.07284	0.00020 0.00002 0.00466	0.68318 0.74643 0.77473
state 1 2 3 4	E_KS -0.40055 -0.20836 -0.04760 -0.04760	E_HF -0.62733 -0.33803 -0.12044 -0.12044	E_GW -0.40035 -0.20833 -0.04295 -0.04300	Sx -0.69022 -0.58997 -0.55914 -0.55914	Re(Sc) 0.22708 0.12970 0.07884 0.07879	Im(Sc) 0.02707 0.00745 0.00053 0.00050	-0.46344 -0.46029 -0.48631 -0.48631	-0.22678 -0.12968 -0.07284 -0.07284	0.00020 0.00002 0.00466 0.00460	0.68318 0.74643 0.77473 0.77383
state 1 2 3 4 5	E_KS -0.40055 -0.20836 -0.04760 -0.04760 0.04899	E_HF -0.62733 -0.33803 -0.12044 -0.12044 0.22993	E_GW -0.40035 -0.20833 -0.04295 -0.04300 0.07841	Sx -0.69022 -0.58997 -0.55914 -0.55914 -0.28110	Re(Sc) 0.22708 0.12970 0.07884 0.07879 -0.14333	Im(Sc) 0.02707 0.00745 0.00053 0.00050 -0.00053	-0.46344 -0.46029 -0.48631 -0.48631 -0.46204	-0.22678 -0.12968 -0.07284 -0.07284 0.18094	0.00020 0.00002 0.00466 0.00460 0.02942	0.68318 0.74643 0.77473 0.77383 0.78220
state 1 2 3 4 5	E_KS -0.40055 -0.20836 -0.04760 -0.04760 0.04899 0.11512	E_HF -0.62733 -0.33803 -0.12044 -0.12044 0.22993 0.30416	E_GW -0.40035 -0.20833 -0.04295 -0.04300 0.07841 0.14384	Sx -0.69022 -0.58997 -0.55914 -0.55914 -0.28110 -0.22532	Re(Sc) 0.22708 0.12970 0.07884 0.07879 -0.14333 -0.15215	Im(Sc) 0.02707 0.00745 0.00053 0.00050 -0.00053 -0.00174	-0.46344 -0.46029 -0.48631 -0.48631 -0.46204 -0.41437	-0.22678 -0.12968 -0.07284 -0.07284 0.18094 0.18905	0.00020 0.00002 0.00466 0.00460 0.02942 0.02872	0.68318 0.74643 0.77473 0.77383 0.78220 0.77830
state 1 2 3 4 5 6 7	E_KS -0.40055 -0.20836 -0.04760 -0.04760 0.04899 0.11512 0.11512	E_HF -0.62733 -0.33803 -0.12044 -0.12044 0.22993 0.30416 0.30416	E_GW -0.40035 -0.20833 -0.04295 -0.04300 0.07841 0.14384 0.14394	Sx -0.69022 -0.58997 -0.55914 -0.55914 -0.28110 -0.22532 -0.22532	Re(Sc) 0.22708 0.12970 0.07884 0.07879 -0.14333 -0.15215 -0.15203	Im(Sc) 0.02707 0.00745 0.00053 0.00050 -0.00053 -0.00174 -0.00165	-0.46344 -0.46029 -0.48631 -0.48631 -0.46204 -0.41437 -0.41437	-0.22678 -0.12968 -0.07284 -0.07284 0.18094 0.18905 0.18905	0.00020 0.00002 0.00466 0.00460 0.02942 0.02872 0.02883	0.68318 0.74643 0.77473 0.77383 0.78220 0.77830 0.77885
state 1 2 3 4 5 6 7 8	E_KS -0.40055 -0.20836 -0.04760 -0.04760 0.04899 0.11512 0.11512 0.26948	E_HF -0.62733 -0.33803 -0.12044 -0.12044 0.22993 0.30416 0.30416 0.49261	E_GW -0.40035 -0.20833 -0.04295 -0.04300 0.07841 0.14384 0.14394 0.29430	Sx -0.69022 -0.58997 -0.55914 -0.55914 -0.28110 -0.22532 -0.22532 -0.15957	Re(Sc) 0.22708 0.12970 0.07884 0.07879 -0.14333 -0.15215 -0.15203 -0.19119	Im(Sc) 0.02707 0.00745 0.00053 0.00050 -0.00053 -0.00174 -0.00165 -0.00829	-0.46344 -0.46029 -0.48631 -0.48631 -0.46204 -0.41437 -0.41437 -0.38270	-0.22678 -0.12968 -0.07284 -0.07284 0.18094 0.18905 0.18905 0.22314	0.00020 0.00002 0.00466 0.00460 0.02942 0.02872 0.02883 0.02483	0.68318 0.74643 0.77473 0.77383 0.78220 0.77830 0.77885 0.77719

k-point	# 1:	0.000000	0.00000	0.00000	0 0.0370	937				
state [E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
ų										
k-point	# 2:	0.000000	0.000000	0.33333	3 0.2962	296				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.40055	-0.62733	-0.40035	-0.69022	0.22708	0.02707	-0.46344	-0.22678	0.00020	0.68318
2	-0.20836	-0.33803	-0.20833	-0.58997	0.12970	0.00745	-0.46029	-0.12968	0.00002	0.74643
3	-0.04760	-0.12044	-0.04295	-0.55914	0.07884	0.00053	-0.48631	-0.07284	0.00466	0.77473
4	-0.04760	-0.12044	-0.04300	-0.55914	0.07879	0.00050	-0.48631	-0.07284	0.00460	0.77383
5	0.04899	0.22993	0.07841	-0.28110	-0.14333	-0.00053	-0.46204	0.18094	0.02942	0.78220
6	0.11512	0.30416	0.14384	-0.22532	-0.15215	-0.00174	-0.41437	0.18905	0.02872	0.77830
7	0.11512	0.30416	0.14394	-0.22532	-0.15203	-0.00165	-0.41437	0.18905	0.02883	0.77885
8	0.26948	0.49261	0.29430	-0.15957	-0.19119	-0.00829	-0.38270	0.22314	0.02483	0.77719
9	0.28052	0.50287	0.31461	-0.16878	-0.17908	-0.00819	-0.39114	0.22236	0.03409	0.78765
10	0.20052	0.30287	0.31401	-0.10070	-0.17500	-0.00019	-0.57114	0.22250	0.05402	0.78705

k-point	:# 1:	0.000000	0.000000	0.00000	0 0.037	937				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
		·								
k-point		0.000000	0.00000							
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.40055	-0.62733	-0.40035	-0.69022	0.22708	0.02707	-0.46344	-0.22678	0.00020	0.68318
2	-0.20836	-0.33803	-0.20833	-0.58997	0.12970	0.00745	-0.46029	-0.12968	0.00002	0.74643
3	-0.04760	-0.12044	-0.04295	-0.55914	0.07884	0.00053	-0.48631	-0.07284	0.00466	0.77473
4	-0.04760	-0.12044	-0.04300	-0.55914	0.07879	0.00050	-0.48631	-0.07284	0.00460	0.77383
5	0.04899	0.22993	0.07841	-0.28110	-0.14333	-0.00053	-0.46204	0.18094	0.02942	0.78220
6	0.11512	0.30416	0.14384	-0.22532	-0.15215	-0.00174	-0.41437	0.18905	0.02872	0.77830
7	0.11512	0.30416	0.14394	-0.22532	-0.15203	-0.00165	-0.41437	0.18905	0.02883	0.77885
8	0.26948	0.49261	0.29430	-0.15957	-0.19119	-0.00829	-0.38270	0.22314	0.02483	0.77719
9	0.28052	0.50287	0.31461	-0.16878	-0.17908	-0.00819	-0.39114	0.22236	0.03409	0.78765

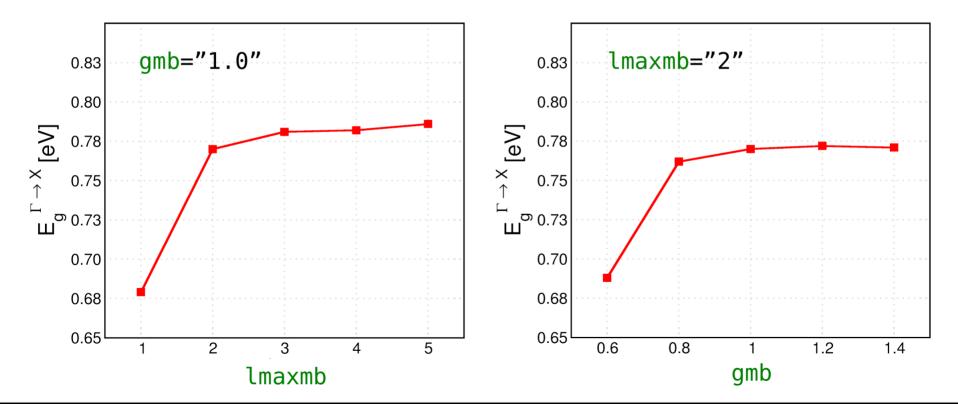
k-point	# 1:	0.00000	0.00000	0.0000	0.0370	937				
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.45134	-0.69966	-0.44986	-0.70105	0.25056	0.03612	-0.45273	-0.24832	0.00149	0.66555
2	-0.01047	-0.07414	-0.00378	-0.56215	0.07225	0.00015	-0.49848	-0.06367	0.00670	0.78017
3	-0.01047	-0.07414	-0.00389	-0.56215	0.07211	0.00013	-0.49848	-0.06367	0.00658	0.77903
4	-0.01047	-0.07414	-0.00389	-0.56215	0.07212	0.00014	-0.49848	-0.06367	0.00659	0.77917
5	0.08067	0.26157	0.10936	-0.25087	-0.14402	-0.00125	-0.43177	0.18090	0.02869	0.77794
6	0.08067	0.26157	0.10933	-0.25087	-0.14409	-0.00127	-0.43177	0.18090	0.02865	0.77846
7	0.08067	0.26157	0.10942	-0.25087	-0.14399	-0.00120	-0.43177	0.18090	0.02875	0.77888
8	0.10651	0.32083	0.14503	-0.34472	-0.16419	-0.00147	-0.55904	0.21432	0.03852	0.76837
9	0.26869	0.47704	0.28745	-0.13571	-0.18444	-0.00703	-0.34406	0.20835	0.01875	0.78431
10	0.27253	0.49322	0.31704	-0.16678	-0.16550	-0.00209	-0.38748	0.22069	0.04451	0.80646
			ųJ							
k-point		0.00000	0.00000							
state	E_KS	E_HF	E_GW	Sx	Re(Sc)	Im(Sc)	Vxc	DE_HF	DE_GW	Znk
1	-0.40055	-0.62733	-0.40035	-0.69022	0.22708	0.02707	-0.46344	-0.22678	0.00020	0.68318
2	-0.20836	-0.33803	-0.20833	-0.58997	0.12970	0.00745	-0.46029	-0.12968	0.00002	0.74643
3	-0.04760	-0.12044	-0.04295	-0.55914	0.07884	0.00053	-0.48631	-0.07284	0.00466	0.77473
4	-0.04760	-0.12044	-0.04300	-0.55914	0.07879	0.00050	-0.48631	-0.07284	0.00460	0.77383
5	0.04899	0.22993	0.07841	-0.28110	-0.14333	-0.00053	-0.46204	0.18094	0.02942	0.78220
6	0.11512	0.30416	0.14384	-0.22532	-0.15215	-0.00174	-0.41437	0.18905	0.02872	0.77830
7	0.11512	0.30416	0.14394	-0.22532	-0.15203	-0.00165	-0.41437	0.18905	0.02883	0.77885
8	0.26948	0.49261	0.29430	-0.15957	-0.19119	-0.00829	-0.38270	0.22314	0.02483	0.77719
9				0 4 6 0 7 0	0 47000	0 00010	0 20444	0 22226	0 00 400	
	0.28052	0.50287	0.31461	-0.16878	-0.17908	-0.00819	-0.39114	0.22236	0.03409	0.78765

Convergence test

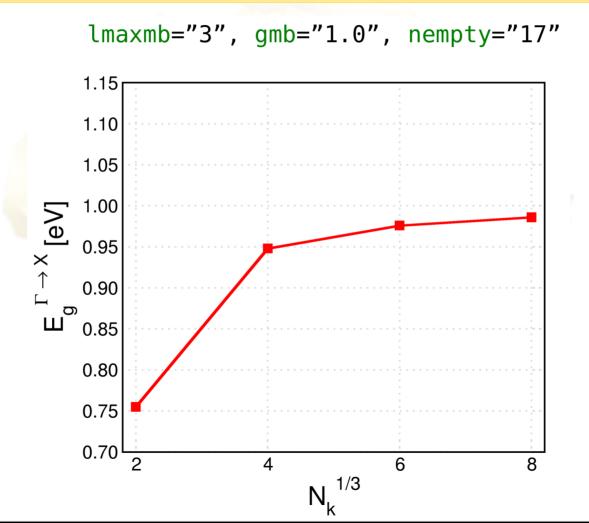
/input/gw/mixbasis/@lmaxmb

/input/gw/mixbasis/@gmb

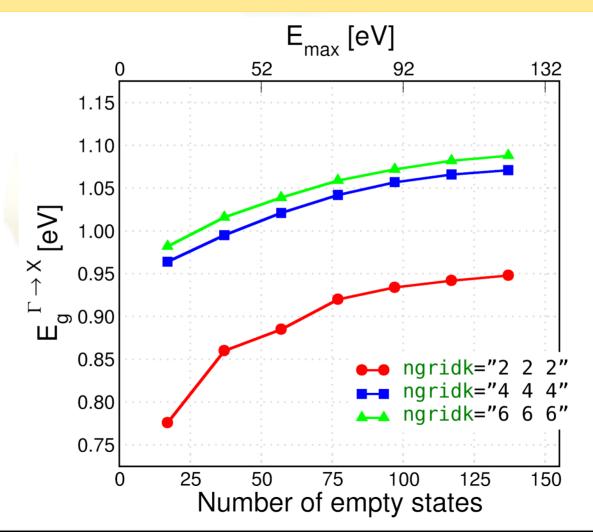
nempty="17", ngridk="2 2 2"



Convergence test



Convergence test



Summary

- GW approximation and implementation in exciting
- Inputs for GW@exciting
- Key factors to run GW in exciting
- Perform the convergence tests



Summary

- GW approximation and implementation in exciting
- Inputs for GW@exciting
- Key factors to run GW in exciting
- Perform the convergence tests
- ► Do not use as a **BLACK BOX**



Summary

- GW approximation and implementation in exciting
- Inputs for GW@exciting
- Key factors to run GW in exciting
- Perform the convergence tests
- ► Do not use as a **BLACK BOX**







Low scaling GW algorithm

$\mathcal{O}(N^3)$

NOMAD Center of Excellence



